

Elusive Neutrons from Nuclei in Effective Field Theory

Silas R. Beane*

**Department of Physics, University of Maryland
College Park, MD 20742-4111*

Abstract. We review recent computations of neutral pion photoproduction and Compton scattering on the deuteron in baryon chiral perturbation theory. Progress in extracting the neutron electric dipole amplitude, which is relevant in neutral pion photoproduction, and the neutron polarizabilities, which are relevant in Compton scattering, is discussed.

INTRODUCTION

The absence of suitable neutron targets in low-energy scattering experiments requires the use of nuclear targets like deuterium and helium in order to extract neutron scattering data. The extent to which neutron data can be reliably extracted depends on how under control the errors are in computing the nuclear corrections to free nucleon motion. Of course precise calculations of hadron processes are possible only where a small dimensionless expansion parameter is identified. This is the main motivation behind the ongoing intense effort to develop a perturbative theory of nuclear interactions [1]. The dimensionless parameters relevant to low energy QCD and therefore to nuclear physics consist of ratios of external momenta to various characteristic energy scales, like the nucleon mass. Effective field theory is the technology which develops a hierarchy of scales into a perturbative expansion of physical observables.

In this paper we describe several recent effective field theory calculations whose objective is to extract neutron properties from nuclear scattering processes in a systematic way. We first discuss a computation of neutral pion photoproduction on the deuteron and its dependence on nucleon parameters. We then describe a calculation of Compton scattering on the deuteron at photon energies of order the pion mass. Here the ultimate objective is to learn about neutron polarizabilities from nuclear Compton scattering. The basic power-counting scheme is reviewed in the first section. In the second section we discuss photoproduction on the deuteron. The third section is dedicated to Compton scattering on the deuteron.

WEINBERG POWER-COUNTING

At energies well below the chiral symmetry breaking scale, $\Lambda_\chi \sim 4\pi f_\pi \sim M \sim m_\rho$, the interactions of pions, photons and nucleons can be described systematically using an effective field theory. This effective field theory, known as chiral perturbation theory (χPT), reflects the observed QCD pattern of symmetry breaking. In QCD the chiral $SU(2)_L \times SU(2)_R$ symmetry is spontaneously broken. Here we are interested in processes where the typical momenta of all external particles is $p \ll \Lambda_\chi$, so we identify our expansion parameter as p/Λ_χ . In QCD $SU(2)_L \times SU(2)_R$ is softly broken by the small quark masses. This explicit breaking implies that the pion has a small mass in the low-energy theory. Since m_π/Λ_χ is then also a small parameter, we have a dual expansion in p/Λ_χ and m_π/Λ_χ . We take Q to represent either a small momentum *or* a pion mass.

In few-nucleon systems, a complication arises in χPT due to the existence of shallow nuclear bound states and related infrared singularities in A -nucleon reducible Feynman diagrams evaluated in the static approximation [2]. The fundamental problem is that nuclear physics introduces a new mass scale, the nuclear binding energy, which is very small compared to a typical hadronic scale like Λ_χ . One way to overcome this difficulty is to adopt a modified power-counting scheme in which χPT is used to calculate an effective potential which generally consists of all A -nucleon irreducible graphs. The S -matrix, which includes all reducible graphs as well, is then obtained through iteration by solving a Lippmann-Schwinger equation [2]. This version of nuclear effective theory is known as the Weinberg formulation. There now exists a competing power-counting scheme in which all nonperturbative physics responsible for the presence of low-lying bound states arises from the iteration of a single operator in the effective theory, while all other effects, including all higher dimensional operators *and* pion exchange, are treated perturbatively [3,4]. This version of the effective theory is known as the Kaplan-Savage-Wise (KSW) formulation. This is relevant here because Compton scattering on the deuteron has been computed to next-to-leading order in the KSW formulation [5]. We will discuss this result and its relation to our calculation. A comprehensive and up-to-date review of nuclear applications of effective field theories can be found in Ref. [1].

It should be noted that typical nucleon momenta inside the deuteron are small—on the order of \sqrt{MB} or m_π , with B the deuteron binding energy—and consequently, *a priori* we expect no convergence problems in the χPT expansion of any low-momentum electromagnetic or pionic probe of the deuteron. Although in principle we could use wavefunctions computed in χPT , we will consider wavefunctions generated using modern nucleon-nucleon potentials [6]. Generally we find that any wavefunction with the correct binding energy gives equivalent results to within the theoretical error expected from neglected higher orders in the chiral expansion. Presumably we are insensitive to short distance components of the wavefunction because we are working at low energies and the deuteron is a large object.

PHOTOPRODUCTION ON THE DEUTERON

A striking example of the power of effective field theory is neutral pion photoproduction on the deuteron at threshold. The $O(Q^4)$ χPT prediction for the electric dipole amplitude in neutral pion photoproduction on the deuteron is [7]

$$\begin{aligned} E_d &= E_d^{ss} + E_d^{tb,3} + E_d^{tb,4} \\ &= (0.36 - 1.90 - 0.25) \times 10^{-3}/m_{\pi^+} \\ &= (-1.8 \pm 0.2) \times 10^{-3}/m_{\pi^+} \end{aligned} \quad (1)$$

where $E_d^{tb,3}$ and $E_d^{tb,4}$ represent $O(Q^3)$ and $O(Q^4)$ three-body corrections, respectively, and

$$\begin{aligned} E_d^{ss} &= \frac{1 + m_\pi/M}{1 + m_\pi/M_d} \left\{ \frac{1}{2} (E_{0+}^{\pi^0 p} + E_{0+}^{\pi^0 n}) \int d^3 p \phi_f^*(\vec{p}) \vec{\epsilon} \cdot \vec{J} \phi_i(\vec{p} - \vec{k}/2) \right. \\ &\quad \left. - \frac{k}{M} \hat{k} \cdot \int d^3 p \hat{p} \frac{1}{2} (P_1^{\pi^0 p} + P_1^{\pi^0 n}) \phi_f^*(\vec{p}) \vec{\epsilon} \cdot \vec{J} \phi_i(\vec{p} - \vec{k}/2) \right\}, \end{aligned} \quad (2)$$

where ϕ represents the deuteron wavefunction, here taken from the Argonne V18 potential, M_d is the deuteron mass and \vec{k} is the photon momentum. The $O(Q^4)$ χPT values for the electric dipole amplitudes of the proton and neutron are [8]

$$E_{0+}^{\pi^0 p} = -1.16 \times 10^{-3}/m_{\pi^+} \quad E_{0+}^{\pi^0 n} = +2.13 \times 10^{-3}/m_{\pi^+}. \quad (3)$$

Note the large value of the neutron electric dipole amplitude. The corresponding neutron cross section is a factor of four larger than the proton cross section, in complete violation of classical intuition. (These are not bona fide predictions since they involve counterterms determined by resonance saturation.) The proton empirical value from MAMI [9] is

$$E_{0+}^{\pi^0 p} = (-1.31 \pm 0.08) \times 10^{-3}/m_{\pi^+}, \quad (4)$$

in agreement with the χPT prediction. In order to test the neutron prediction we must consider the deuteron. The SAL empirical value [10] for the deuteron is

$$E_d^{exp} = (-1.45 \pm 0.09) \times 10^{-3}/m_{\pi^+} \quad (5)$$

and therefore overlaps with the χPT prediction within 1.5σ . One may wonder about the sensitivity of the deuteron electric dipole amplitude to the neutron contribution. For instance, if we set $E_{0+}^{\pi^0 n} = 0$ the χPT prediction becomes $E_d = -2.6$, completely at odds with the experimental value. This result is a striking confirmation of the large χPT prediction for the neutron. It is interesting that models miss the large chiral loop effects and therefore predict a much smaller neutron electric dipole amplitude. See Fig. 1.

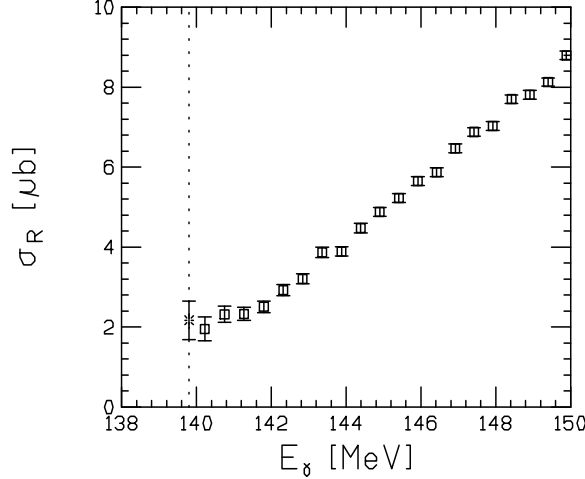


FIGURE 1. SAL data versus χPT prediction (star).

DEUTERON COMPTON SCATTERING

Nucleon Compton scattering has been studied in χPT in Ref. [11], where the following results for the polarizabilities were obtained to order Q^3 :

$$\begin{aligned}\alpha_p &= \alpha_n = \frac{5e^2 g_A^2}{384\pi^2 f_\pi^2 m_\pi} = 12.2 \times 10^{-4} \text{ fm}^3; \\ \beta_p &= \beta_n = \frac{e^2 g_A^2}{768\pi^2 f_\pi^2 m_\pi} = 1.2 \times 10^{-4} \text{ fm}^3.\end{aligned}\tag{6}$$

Here we have used $g_A = 1.26$ for the axial coupling of the nucleon, and $f_\pi = 93$ MeV as the pion decay constant. Note that the polarizabilities are *predictions* of χPT at this order. The $O(Q^3)$ χPT predictions diverge in the chiral limit because they arise from pion loop effects.

Recent experimental values for the proton polarizabilities are [12]¹

$$\begin{aligned}\alpha_p + \beta_p &= 13.23 \pm 0.86_{-0.49}^{+0.20} \times 10^{-4} \text{ fm}^3, \\ \alpha_p - \beta_p &= 10.11 \pm 1.74_{-0.86}^{+1.22} \times 10^{-4} \text{ fm}^3,\end{aligned}\tag{7}$$

where the first error is a combined statistical and systematic error, and the second set of errors comes from the theoretical model employed. These values are in good agreement with the χPT predictions.

On the other hand, the neutron polarizabilities are difficult to obtain experimentally and so the corresponding χPT prediction is not well tested. One way to extract neutron polarizabilities is to consider Compton scattering on nuclear targets. Consider coherent photon scattering on the deuteron. The cross section in the forward direction naively goes as:

¹⁾ These are the result of a model-dependent fit to data from Compton scattering on the proton at several angles and at energies ranging from 33 to 309 MeV.

$$\left. \frac{d\sigma}{d\Omega} \right|_{\theta=0} \sim (f_{Th} - (\alpha_p + \alpha_n)\omega^2)^2. \quad (8)$$

The sum $\alpha_p + \alpha_n$ may then be accessible via its interference with the dominant Thomson term for the proton, f_{Th} [13]. This means that with experimental knowledge of the proton polarizabilities it may be possible to extract those for the neutron. Coherent Compton scattering on a deuteron target has been measured at $E_\gamma = 49$ and 69 MeV by the Illinois group [14]. An experiment with tagged photons in the energy range $E_\gamma = 84.2 - 104.5$ MeV is under analysis at Saskatoon [15].

Clearly the amplitude for Compton scattering on the deuteron involves mechanisms other than Compton scattering on the individual constituent nucleons. Hence, extraction of nucleon polarizabilities requires a theoretical calculation of Compton scattering on the deuteron that is under control in the sense that it accounts for *all* mechanisms to a given order in a systematic expansion in a small parameter.

In the remainder of this paper we will review a recent computation of Compton scattering on the deuteron for incoming photon energies of order 100 MeV in the Weinberg formulation [16]. As in the computation of the electric dipole amplitude in photoproduction, baryon χPT is used to compute an irreducible scattering kernel (here to order Q^3) which is then sewn to external deuteron wavefunctions.

In Figures 2 and 3 we display our results at 69 and 95 MeV. For comparison we have included the calculation at $O(Q^2)$ where the γN T -matrix in the single-scattering contribution is given by the Thomson term on a single nucleon. It is remarkable that to $O(Q^3)$ no unknown counterterms appear. All contributions to the kernel are fixed in terms of known pion and nucleon parameters such as m_π , g_A , M , and f_π . Thus, to this order χPT makes *predictions* for Compton scattering.

The curves show that the correction from the $O(Q^3)$ terms gets larger as ω is increased, as was to be expected. Indeed, while at lower energies corrections are relatively small, in the 95 MeV results the correction to the differential cross section from the $O(Q^3)$ terms is of order 50%, although the contribution of these terms to the *amplitude* is of roughly the size one would expect from the power-counting: about 25%. Nevertheless, it is clear, even from these results, that this calculation must be performed to $O(Q^4)$ before conclusions can be drawn about polarizabilities from data at photon energies of order m_π . This is in accord with similar convergence properties for the analogous calculation for threshold pion photoproduction on the deuteron [7].

We have also shown the Illinois data points at 69 MeV [14]. Statistical and systematic errors have been added in quadrature. The agreement of the $O(Q^3)$ calculation with the 69 MeV data is very good, although only limited conclusions can be drawn, given that there are only two data points, each with sizeable error bars.

Although nominally the domain of validity of the Weinberg formulation extends well beyond the threshold for pion production, the power-counting fails at low

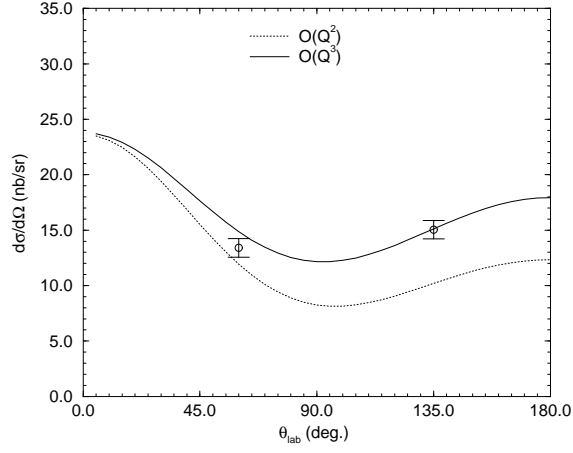


FIGURE 2. Results of the $O(Q^2)$ (dotted line) and $O(Q^3)$ (solid line) calculations at a photon laboratory energy of 69 MeV.

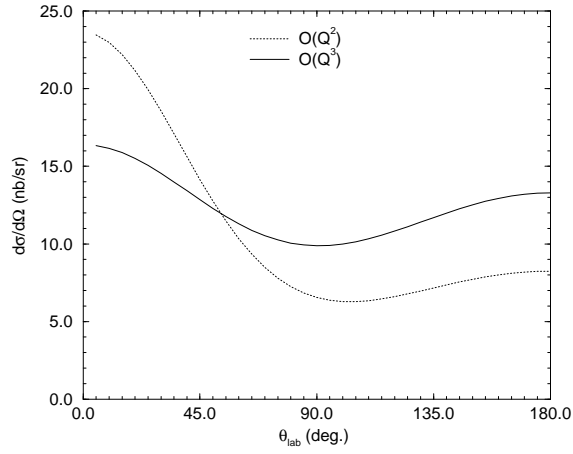


FIGURE 3. Results of the $O(Q^2)$ (dotted line) and $O(Q^3)$ (solid line) calculations at a photon laboratory energy of 95 MeV.

energies well before the Thomson limit is reached. By comparing $O(Q^4)$ and $O(Q^3)$ contributions, it is straightforward to show that χPT breaks down when

$$\frac{|\vec{p}|^2}{\omega M} \sim 1. \quad (9)$$

Here \vec{p} is a typical nucleon momentum inside the deuteron and ω is the photon energy. Since our power-counting is predicated on the assumption that all momenta are of order m_π , we find that power-counting is valid in the region

$$\frac{m_\pi^2}{M} \ll Q \ll \Lambda_\chi. \quad (10)$$

Therefore, in the region $\omega \sim B$ the Weinberg power-counting is not valid, since the external probe momentum flowing through the nucleon lines is of order Q^2/M , rather than order Q . It is in this region that the Compton low-energy theorems are derived. Therefore our power-counting will not recover those low-energy theorems. Of course the upper bound on the validity of the effective theory should increase if the Δ -resonance is included as a fundamental degree of freedom.

In Ref. [5] Compton scattering on the deuteron was computed to the same order discussed here, one order beyond leading non-vanishing order. An advantage of KSW power-counting is that the effective field theory moves smoothly between $Q < B$ and $Q > B$. KSW power-counting is valid for nucleon momenta $Q < \Lambda_{NN} \sim 300$ MeV. Thus in the KSW formulation deuteron polarizabilities and Compton scattering up to energies $\omega < \Lambda_{NN}^2/M \sim 90$ MeV can be discussed in the same framework. Here we are interested mostly in the region $\omega \sim m_\pi$, and so we regard ourselves as being firmly in the second regime. We stress that the value of Λ_{NN} is uncertain; it is conceivable that $\Lambda_{NN} \sim 500$ MeV in which case the range of the KSW formulation would extend well beyond pion production threshold.

Comparing to the calculations of deuteron Compton scattering in the KSW formulation of effective field theory [5], we see that the result of Ref. [5] is significantly lower than those presented here at both 49 and 69 MeV. At 49 MeV (not shown here) the agreement of Ref. [5]'s calculation with the data is better than ours. Presumably this is partly because 49 MeV is at the lower end of the domain of applicability of the Weinberg formulation. At 69 MeV our calculation does a slightly better job of reproducing the (two) data points available. The qualitative agreement among these calculations is a reflection of the similarities of mechanisms involved. Ours is however the only calculation to incorporate the full single-nucleon amplitude instead of its polarizability approximation. (Note however that in Ref. [5], the first corrections to the polarizability approximation of the pion graphs are included and found to be very small.) Our tendency to higher relative cross sections in the backward directions is at least in part due to this feature.

In order to test the sensitivity of our calculation to higher-order effects we added a small piece of the $O(Q^4)$ amplitude for Compton scattering off a single nucleon. As one would expect, we find that the cross section at 95 MeV is much more

sensitive to these $O(Q^4)$ terms than the cross section at 49 MeV. In our view, a full $O(Q^4)$ calculation in χPT is necessary if any attempt is to be made to extract the neutron polarizability from the Saskatoon data within this framework.

ACKNOWLEDGMENTS

I thank Veronique Bernard, Harry Lee, Mané Malheiro, Ulf Meißner, Dan Phillips and Ubi van Kolck for enjoyable collaborations and Ubi van Kolck for a critical reading of the manuscript. This research was supported by DOE grant DE-FG02-93ER-40762 (DOE/ER/40762-193, UMPP#00-016).

REFERENCES

1. R. Seki, U. van Kolck and M. J. Savage eds., *Nuclear Physics with Effective Field Theory*, World Scientific, 1998; U. van Kolck, [nucl-th/9902015](#).
2. S. Weinberg, *Phys. Lett. B.* **251**, 288 (1990); *Nucl. Phys. B.* **363**, 3 (1991).
3. U. van Kolck, in *Mainz 1997, Chiral Dynamics: Theory and Experiment*, ed. A. Bernstein *et al*, Springer-Verlag, 1998, [hep-ph/9711222](#); *Nucl. Phys. A.* **645**, 273 (1999), [nucl-th/9808007](#).
4. D. B. Kaplan, M. J. Savage, and M. B. Wise, *Phys. Lett. B.* **424**, 390 (1998), [nucl-th/9801034](#); *Nucl. Phys. B.* **534**, 329 (1998), [nucl-th/9802075](#).
5. J.-W. Chen, H. W. Griesshammer, M. J. Savage and R. P. Springer, *Nucl. Phys. A.* **644**, 221 (1998), [nucl-th/9806080](#); *Nucl. Phys. A.* **644**, 245 (1998), [nucl-th/9809023](#); J.-W. Chen, *Nucl. Phys. A* **653**, 375 (1999), [nucl-th/9810021](#).
6. S. Weinberg, *Phys. Lett. B.* **295**, 114 (1992).
7. S. R. Beane, C. Y. Lee, and U. van Kolck, *Phys. Rev. C.* **52**, 2914 (1995), [nucl-th/9506017](#); S.R. Beane, V. Bernard, T.S.H. Lee, Ulf-G. Meißner and U. van Kolck, *Nucl. Phys.* **618**, 381 (1997), [hep-ph/9702226](#).
8. V. Bernard, N. Kaiser and Ulf-G. Meißner, *Phys. Lett. B.* **378**, 337 (1996), [hep-ph/9512234](#).
9. M. Fuchs *et al*, *Phys. Lett. B.* **368**, 20 (1996).
10. J. Bergstrom *et al*, *Phys. Rev. C.* **57**, 3202 (1998).
11. V. Bernard, N. Kaiser, and Ulf-G. Meißner, *Phys. Rev. Lett.* **67**, 1515 (1991); *Nucl. Phys. B.* **383**, 442 (1992); V. Bernard, N. Kaiser, J. Kambor, and Ulf-G. Meißner, *Nucl. Phys. B.* **388**, 315 (1992).
12. J. Tonnison, A. M. Sandorfi, S. Hoblit, and A. M. Nathan, *Phys. Rev. Lett.* **80**, 4382 (1998), [nucl-th/9801008](#).
13. D. Drechsel, *et al*, in *Mainz 1997, Chiral Dynamics: Theory and Experiment*, ed. A. Bernstein *et al*, Springer-Verlag, 1998, p. 264, [nucl-th/9712013](#).
14. M. Lucas, Ph. D. thesis, University of Illinois, unpublished (1994).
15. D. Hornidge, private communication; G. Feldman, private communication.
16. S.R. Beane, M. Malheiro, D.R. Phillips and U. van Kolck, [nucl-th/9905023](#).